BIRZEIT UNIVERSITY

Mathematics Department

Calculus I -- Math1411

Second Exam **Time: 90 Minutes** First Semester 2021 – 2022

Name: Key B Number: Section:

- Write your full name and your number
- Choose your section from table below
- Turn off your mobile
- Calculator is not allowed
- The exam has 9 different pages. Answer all questions

Section	Instructor	Day	Time	Room
1	Farah Omar	T	14:15 - 15:05	SCI120
2	Areej Awawdah	S	08:00 - 08:50	SCI240
3	Muna Abu Alhalawa	R	09:00 - 09:50	S.Abdulhadi380
4	Areej Awawdah	R	11:25 - 12:15	S.Abdulhadi380
5	Ayah Sharsheer	S	11:25 - 12:15	Al-Juraysi002
6	Muna Abu Alhalawa	T	09:00 - 09:50	S.Abdulhadi380
7	Batool Raddad	T	14:15 - 15:05	SCI240
8	Ayah Sharsheer	S	08:00 - 08:50	S.Abdulhadi380
9	Farah Omar	R	13:00 - 13:50	SCI120
10	Ayah Sharsheer	S	13:00 - 13:50	O.Abdulhadi051
11	Ayah Sharsheer	T	08:00 - 08:50	S.Abdulhadi380
12	Ayah Sharsheer	S	09:00 - 09:50	SCI120
13	Alaeddin Elayyan	R	10:00 - 10:50	S.Abdulhadi380
14	Batool Raddad	S	08:00 - 08:50	Al-Juraysi002
15	Farah Omar	T	09:00 - 09:50	SCI240
16	Batool Raddad	R	14:15 - 15:05	SCI120
17	Ayah Sharsheer	T	10:00 - 10:50	SCI240
18	Farah Omar	S	09:00 - 09:50	O.Abdulhadi051
19	Areej Awawdah	R	08:00 - 08:50	SCI240
20	Batool Raddad	S	13:00 - 13:50	O.Abdulhadi052
21	Ayah Sharsheer	W	14:15 - 15:05	S.Abdulhadi380

Question One (21 points) Circle the most correct answer:

1. $\int_1^9 \frac{1}{2x} \log_3^{x^2} \, dx =$

   (a) $\ln 2$

   (b) $\ln 4$

   (c) $\ln 3$

   (d) $\ln 9$

2. If $f(x) = 2^{\sqrt{x}} + \ln 2^x$, then $f'(1) =$

   (a) $\ln 4$

   (b) $\ln 8$

   (c) $\ln 32$

   (d) $\ln 64$

3. If $e^{2\ln 3} - 27^{\log_3^2} = \frac{x}{2}$ then $x =$

   (a) 2

   (b) 4

   (c) 1

   (d) 3

4. The volume of the solid generated by revolving the region bounded by $y = x^2$, $x = 1$, $y = 0$ **about the** x**-axis** is

   (a) $\frac{\pi}{5}$

   (b) $\frac{\pi}{4}$

   (c) $\frac{\pi}{3}$

   (d) $\frac{\pi}{2}$

5. If $y = (\ln x)^{x^2}$, then $y'(e) =$

(a) 1
(b) e
(c) 2
(d) 2e

6. If $f'(x) = \cos x$ for $x \in (0, \pi)$, then $\frac{df^{-1}}{dx}$ at $x = f(\frac{\pi}{3})$ is

(a) 0.5
(b) 0
(c) 1
(d) 2

7. A possible curve $y = f(x)$ whose length $\int_0^1 \sqrt{2+x}\, dx$ and passes through the point $(0, -\frac{1}{3})$ is

(a) $y = \frac{3}{2}(x+1)^{\frac{2}{3}} + 1$
(b) $y = \frac{3}{2}(x+1)^{\frac{2}{3}} - 1$
(c) $y = \frac{2}{3}(x+1)^{\frac{3}{2}} - 1$
(d) $y = \frac{2}{3}(x+1)^{\frac{3}{2}} + 1$

8. $\lim_{x \to 0^+} (\ln(\tan x) - \ln x) =$

(a) 0
(b) e
(c) 1
(d) ∞

9. $\int_1^{e^2} \frac{\cos(\ln x)}{x}\, dx =$

(a) $\sin 1$
(b) $\sin 2$
(c) $\cos 1$
(d) $\cos 2$

10. If $y = -\left(\sqrt{2}\right)^{\ln(\cot x)}$, then $y'(\frac{\pi}{4}) =$

(a) $\sqrt{2}\ln 2$
(b) $\ln\sqrt{2}$
(c) $\ln 2$
(d) $\ln 4$

11. $\int_0^{\sqrt[3]{2}} \ln 4 \; e^{\ln 3+\ln x^2} \; 2^{x^3} dx =$

(a) 2
(b) 4
(c) 6
(d) 8

12. $\lim_{x\to 0^+} x^{\sqrt{x}} =$

(a) 0
(b) e
(c) ∞
(d) 1

13. $\lim_{x\to 0} \frac{\ln(\cos^2 x)}{x^2} =$

(a) 0
(b) 2
(c) -1
(d) ∞

14. If the half-life time of a radioactive material is $\ln 4$ years, then the time required for this material to lose 75% of it's initial amount is

(a) $\ln 2$ years
(b) $\ln 4$ years
(c) $\ln 8$ years
(d) $\ln 16$ years

Question Two (10points) Given the function $f(x) = 1 - \sqrt{x}$

(1) Find $f^{-1}(x)$

(1) $y = 1 - \sqrt{x} \Rightarrow \sqrt{x} = 1 - y$

(1) $x = (1-y)^2 \Rightarrow y = (1-x)^2 \Rightarrow f^{-1}(x) = (1-x)^2$

(2) Find **range** of $f^{-1}(x)$

(1) $R(f^{-1}) = D(f) = [0, \infty)$

(3) Find $\frac{df^{-1}}{dx}$ at $x = -1$

(1) $\frac{df^{-1}}{dx} = -2(1-x)$

(1) $\frac{df^{-1}}{dx}\Big|_{x=-1} = -2(1--1) = -4$

or $\frac{df^{-1}}{dx}\Big|_{x=-1} = \frac{1}{f'(f^{-1}(-1))}$

$= \frac{1}{f'(4)} = \frac{1}{-\frac{1}{4}} = -4$

where $f'(x) = -\frac{1}{2\sqrt{x}}$

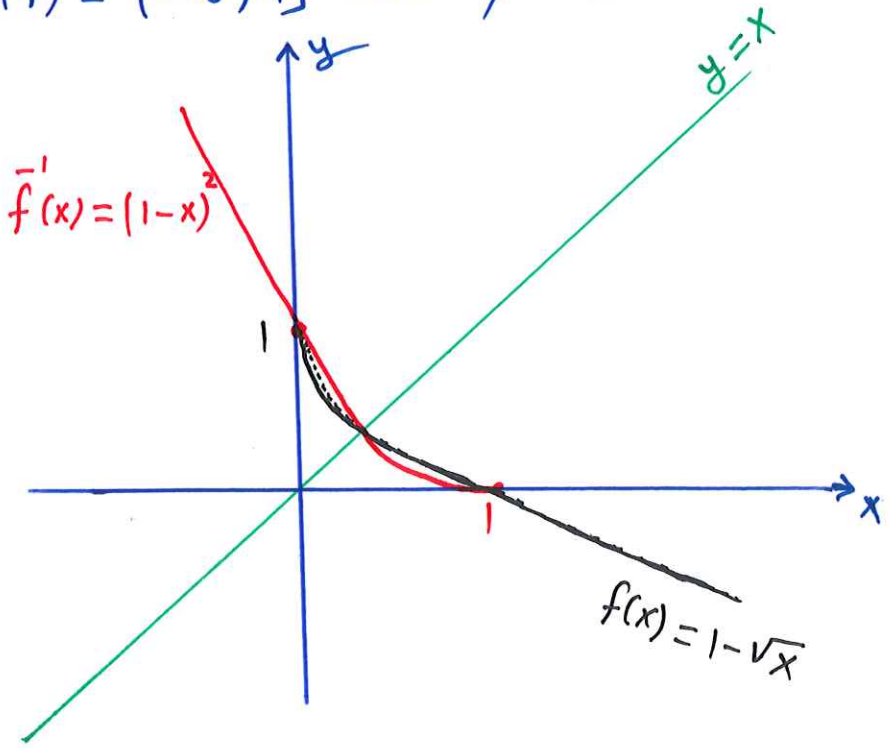
(4) Find $f^{-1}(2)$

(1) Undefined

(1) since $D(f^{-1}) = R(f) = (-\infty, 1]$ and $2 \notin D(f^{-1})$

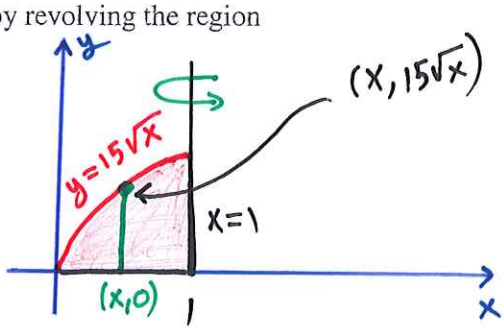
(5) Sketch $f(x)$ and $f^{-1}(x)$

(3)



Question Three (1[points) Answer the following

(a) **Using Shell Method**, find the volume of the solid generated by revolving the region bounded by $y = 15\sqrt{x},\ y = 0,\ x = 1$ **about the line** $x = 1$



(1) $V = 2\pi \int_a^b \left(\text{shell radius}\right)\left(\text{shell height}\right) dx$

(1) $= 2\pi \int_0^1 (1-x)\,(15\sqrt{x})\, dx$

(1) $= 2\pi \left[(15)\left(\frac{2}{3}\right)x^{\frac{3}{2}} - (15)\left(\frac{2}{5}\right)x^{\frac{5}{2}}\right]\Big|_0^1$

(1) $= 8\pi$

(b) Find the **arc length** of the curve $f(x) = \ln(\cos x)$ from $x = 0$ to $x = \frac{\pi}{3}$

(1) $L = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$

(1) $= \int_0^{\frac{\pi}{3}} \sqrt{1 + \left(\frac{\sin x}{\cos x}\right)^2}\, dx$

(1) $= \int_0^{\frac{\pi}{3}} \sqrt{1 + \tan^2 x}\, dx$

(1) $= \int_0^{\frac{\pi}{3}} \sqrt{\sec^2 x}\, dx$

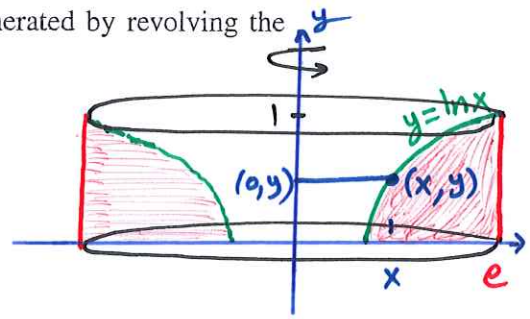
(1) $= \int_0^{\frac{\pi}{3}} \sec x\, dx$ since $\sec x > 0$ on $[0, \frac{\pi}{3}]$

(1) $= \ln|\sec x + \tan x|\Big|_0^{\frac{\pi}{3}}$

(1) $= \ln(2 + \sqrt{3})$

Question Four (9 points) Answer the following

(a) **Using Washer Method**, find the volume of the solid generated by revolving the region bounded by $y = \ln x,\ y = 0,\ x = e$ **about the** y**-axis**



(1) $V = \pi \int_c^d [R^2(y) - r^2(y)]\, dy$

(1) $= \pi \int_0^1 [(e)^2 - (e^y)^2]\, dy$

(1) $= \pi \left[e^2 y - \frac{1}{2} e^{2y} \right]\Big|_0^1$

(1) $= \frac{\pi}{2}(e^2 + 1)$

$R(y) = e$

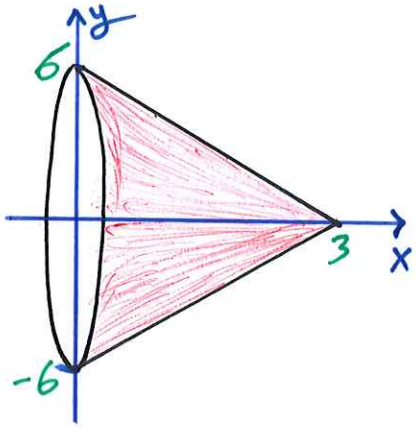
$r(y) = \Delta x = x_2 - x_1$

(1) $= x - 0$

$= x$

$= e^y$

(b) Find the **area of the surface** generated by revolving the line $y = 6 - 2x,\ 0 \le x \le 3$ **about** x**-axis**



(1) $S = 2\pi \int_a^b f(x) \sqrt{1 + (f'(x))^2}\, dx$

(1) $= 2\pi \int_0^3 (6 - 2x) \sqrt{1 + (-2)^2}\, dx$

(1) $= 2\sqrt{5}\,\pi \left[6x - x^2 \right]\Big|_0^3$

(1) $= 18\sqrt{5}\,\pi$

Good Luck